Vilnius University
Institute of Applied Mathematics
European Social Fund (project No 09.3.3-LMT-K-712-17-0003) under
grant agreement with the Research Council of Lithuania (LMTLT)

Multiscale Modeling & Methods

Vilnius, Lithuania, October 24-26, 2022
Workshop

Abstracts

Invited speakers:
F. Chardard (France)
A. Gaudiello (Italy)
I. Pažanin (Croatia)
R. Stavre (Romania)

Organizers: K. Kaulakytė (Lithuania)
G. Panasenko (France, Lithuania)
K. Pileckas (Lithuania)

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Numerical solution of an asymptotic model for viscous fluids in a thin tubes network in the transient regime

Frédéric Chardard

Jean Monnet University, France

A non-stationary flow in a network of thin tubes is considered. Its one-dimensional approximation was proposed in a paper by G.Panasenko and K.Pileckas [1]. It consists of a set of equations with weakly singular kernels, on a graph, for the macroscopic pressure. A new scheme is proposed. It decouples the macroscopic problem on the graph from the problem in the cross-section. On the graph, a finite difference scheme is used to solve the Lax-Milgram problem, for the transient and periodic cases. In the cross-section, we use the the smoothing properties of the heat equation and boundary layer theory to prove that the weakly singular kernels can be be suitably approximated with a finite difference scheme. Numerical results are compared to the direct numerical solution of the full dimension Navier-Stokes equations.

References

[1] G. P. Panasenko, K. Pileckas, Flows in a tube structure: equation on the graph, *Journal of Mathematical Physics* **55** (2014), 081505.

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A spectral problem for the Laplacian in joined thin films

Antonio Gaudiello

Università degli studi della Campania Luigi Vanvitelli Dipartimento di Matematica e Fisica: Caserta, Italy

We consider a $3d$ multi-structure composed of two joined perpendicular thin films: a vertical one with small thickness h_n^a and a horizontal one with small thickness h_n^b. We study the asymptotic behavior, as h_n^a and h_n^b tend to zero, of an eigenvalue problem for the Laplacian defined on this multi-structure. We shall prove that the limit problem depends on the value $q = \lim_n \dfrac{h_n^b}{h_n^a}$. Precisely, we pinpoint three different limit regimes according to q belonging to $]0, +\infty[$, q equal to $+\infty$, or q equal to 0. We identify the limit problems and we also obtain H^1-strong convergence results.

It is a joint work with Delfina Gómez and Maria-Eugenia Pérez-Martínez (Universidad de Cantabria, Santander, Spain).

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Asymptotic Analysis and Numerical Simulations in the heart and in vessels[1]

Grigory Panasenko

Institute Camille Jordan UMR CNRS 5208, University Jean Monnet, Saint-Etienne, France and Institute of Applied Mathematics, Vilnius University, Lithuania

The talk briefly presents the results of asymptotic analysis for non-Newtonian flows in thin tube structures (see [1]). The computation of the leading term of the solution is related to the equation on the graph, which is an elliptic nonlinear problem. We introduce a numerical method to solve the equation on the graph and apply it to the realistic network of vessels. Another numerical simulation is performed in collaboration with the medical part of the team (Prof. Audrius Aidietis and Sigita Aidietienė) in order to detect the stagnation zones in the left atrium appendage of the heart. The results of the numerical part are obtained by the team at Vilnius University: Oleg Ardatov, Sergejus Borodinas, Kristina Kaulakytė, Nikolajus Kozulinas, Grigory Panasenko, Konstantinas Pileckas, Vytenis Šumskas.

References

[1] G. Panasenko, K.Pileckas, and B.Vernescu, Steady state non-Newtonian flow with strain rate dependent viscosity in thin tube structure with no slip boundary condition, *Mathematical Modelling of Natural Phenomena* **17** (2022), 36pp. www.mmnp-journal.org (open access).

[1]This reaearch work has received funding from European Social Fund (project No 09.3.3-LMT-K-712-17-0003) under grant agreement with the Research Council of Lithuania (LMTLT).

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Inertia and roughness-induced effects on the fluid flow through a corrugated domain

Igor Pažanin

Department of Mathematics, Faculty of Science, University of Zagreb, Croatia

We consider an incompressible viscous fluid flowing through a domain with rough wall. Motivated by the applications, we assume the periodicity of the roughness in the longitudinal direction and that the flow is governed by the prescribed pressure drop. The aim is to investigate the effects of the corrugated boundary on the fluid flow by taking into account the effects of inertia as well. Employing the average boundary roughness as the small parameter, we use boundary-layer analysis to derive a higher-order asymptotic approximation of the flow. We propose the new formula for the Darcy-Weisbach friction coefficient and also present the results on the MHD flow through a porous medium.
This is a joint work with Eduard Marušić-Paloka (University of Zagreb).

References

[1] E. Marušić-Paloka, I. Pažanin, Effects of boundary roughness and inertia on the fluid flow through a corrugated pipe and the formula for the Darcy-Weisbach friction coefficient, *International Journal of Engineering Science* **152** (2020), 103293, pp. 1-13.

[2] E. Marušić-Paloka, I. Pažanin, Inertia and roughness-induced effects on the porous medium flow through a corrugated channel, *Transport in Porous Media* **134** (2020), 621–633.

[3] E. Marušić-Paloka, I. Pažanin, A note on the MHD flow in a porous channel, *Theoretical and Applied Mechanics* **49** (1) (2022), 49–60.

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Variational and numerical analysis for a thermal fluid-structure interaction problem

Alexandra-Roxana Ciorogar and Ruxandra Stavre

Institute of Mathematics of the Romanian Academy, Romania

The purpose of this work is to provide a complete study for a thermal fluid-elastic structure interaction problem related to the blood flow through a vessel. The interaction between a viscous fluid and an elastic structure when thermal effects are taken into account is modeled by means of a nonstationary nonlinear coupled system. After linearization and introduction of dimensionless expressions we define a weak solution to the mathematical model and we prove its existence, uniqueness and regularity by means of a variational problem.

Since this problem does not provide enough regularity in the elastic domain, we approximate it with a family of variational viscoelastic problems, depending on a small parameter ε. The viscoelastic problems contain a small additional term that corresponds to the regularity "uncovered" by the initial variational problem. This approximation is justified by an error estimate theorem, followed by a convergence result.

We study next the discretization of an arbitrary viscoelastic problem from the family by means of a finite difference method in time and finite element method in space. The additional viscoelastic term allows us to establish suitable estimates for the solution to the numerical scheme associated with a viscoelastic problem and these estimates are used for obtaining stability properties. The last theoretical result is represented by the convergence of the numerical scheme to the corresponding viscoelastic problem.

Finally, some numerical simulations are performed in order to highlight the changes of the fluid velocity with respect to different characteristics of the interaction problem.